

Phase diagram of the pairing symmetry in two-dimensional strong-coupling superconductors

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Abstract

Two-dimensional Eliashberg equations have been solved by use of a mixed interaction with s - and d -channels. It is discussed what kind of pairing symmetry of the superconducting state can be realized when the channel mixing parameters and the band-filling are varied. By changing the mixing parameters and varying the chemical potential μ between zero- and half-filling, the pairing symmetry is determined and summarized into a phase diagram of the symmetry. It is revealed that there is an effective threshold in μ for the appearance of the d -wave superconductivity regardless of the strength of the d -channel interaction. It is also shown that, although the s -wave superconductivity can occur for any value of μ if the s -channel interaction is mixed sufficiently, the d -wave superconductivity has the advantage of achieving a high T_c over the s -wave one, once it occurs.

Keywords: two-dimensional strong-coupling superconductor, pairing symmetry, mixed interaction of s - and d -wave, band-filling, high- T_c

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1 Introduction

Since the discovery of the high T_c copper oxide superconductors (the cuprates), much attention has been directed to the symmetry of the Cooper-pair in the cuprates. This is partly because we have not yet found a decisive origin of the superconductivity in cuprates, and it is expected that the decision of the symmetry of the superconducting order parameter would illuminate this issue. Among candidates of the pairing symmetry, much of discussions have been focused on the s -wave and the d -wave symmetry, and it has been a central interest which of the two occurs really in cuprates. A lot of experimental investigations have been done with regard to this subject [1]. Their results, however, have not focused decisively on one side of the two. Although recent experiments seem to suggest dominantly the $d_{x^2-y^2}$ symmetry, the results are still controversial even in the new experiments using the Josephson-junction which intend to measure the phase of the order parameter; some led to results taking the side of the $d_{x^2-y^2}$ symmetry [2] while others showed results in favor of the s -wave symmetry [3].

On the theoretical side, there seems to be a general feeling that the realization of the $d_{x^2-y^2}$ pairing state in the cuprates rules out the electron-phonon interaction as the origin of the superconductivity while that of the s -wave pairing state rules out the antiferromagnetic (AFM) spin-fluctuation exchange mechanism and supports the electron-phonon interaction as a main origin. Of course, there have been some works against this common feeling. A recent work [4], for example, has shown that, even if the AFM spin-fluctuation in a CuO_2 layer is responsible for the superconductivity in the cuprates, the s -wave pairing state can be stabilized for a bilayer system of CuO_2 planes by virtue of the existence of strong AFM spin correlations between the constituent CuO_2 layers. On the other hand, some work has predicted the d -wave superconductivity to occur even due to the electron-phonon interaction by starting from an effective single-band Hubbard-type Hamiltonian which includes both the strong electron-electron repulsion and the electron-phonon coupling due to oxygen vibrational modes in CuO_2 planes [5]. An other work showed also that the electron-phonon interaction leads to the d -wave superconductivity based on the “two-story-house-model” which assumes the antiferromagnetically correlated Cu spins as a background of itinerant carriers [6]. These few examples tell us that the pairing symmetry does not correspond uniquely to the pairing mechanism and their realization depends also on the model Hamiltonian which is assumed to describe the electronic state; one-band or multi-band, one-layer or coupled-layers, the strong electronic correlation being taken into account or not, background AFM spin correlation being considered or not and so on. The band-filling as well as the type of the interaction responsible for the superconductivity clearly affects what kind of the pairing symmetry is realized.

In such an entangled situation, it would be necessary for us to simplify the problem and to start with minimal ingredients. As a first step, we restrict ourselves to a two-dimensional CuO_2 layer but do not restrict ourselves to any particular mechanism of the superconductivity. To start with, we address the following simple questions: When a pairing interaction composed of the s -wave and the d -wave channel is introduced into the carriers in a CuO_2 plane, what kind of pairing symmetry of the superconducting state is realized? How does this result depend on the band-filling and the mixing ratio between different channels in the interaction?

This question, however, is not quite new. Lenck et al., on the basis of almost the same

idea but specifying the origins of the interactions to the exchanges of phonons and AFM spin-fluctuations, have examined this problem by solving the Eliashberg equations for carriers described by a two-dimensional tight-binding band [7]. They gave results, however, only for several values of the mixing ratio and the band-filling. The main purpose of this paper is to make a $\mu - s$ phase diagram for the pairing symmetry surviving in a two-dimensional superconductor, where s denotes the mixing parameter of the interaction of s -wave symmetry and μ is the chemical potential which is related to the band-filling of the carriers. Assuming a simple two-dimensional nearest-neighbor tight-binding band, we solve two-dimensional Eliashberg equations by moving thoroughly the mixing parameter s and the chemical potential μ . Such a detailed $\mu - s$ phase diagram for the surviving pairing symmetry has not yet been given by others. The overall tendency of our results coincides with the previous one[7] given by others in a specified region of μ . We calculate, further, the superconducting critical temperature T_c in the realized symmetry state as a function of the mixing parameter s and the chemical potential μ .

The plan of this paper is as follows. In Section 2, our model for the interaction is introduced and a brief survey of our framework of the calculation is given. The essential ingredients for the Eliashberg equations are also presented. In Section 3, the resulting phase-diagrams for the pairing symmetry are discussed. Characteristic difference between the behaviors of T_c in different symmetry states (the s -wave and the d -wave superconducting state) is also pointed out. In the last section, the results of this paper are summarized.

2 Formalism

We start from writing down the two-dimensional Eliashberg equations on the imaginary-axis for carriers with a band energy $\epsilon_{\mathbf{k}}$ in the form of coupled equations for the renormalization function $Z(\mathbf{k}, i\omega_n)$, the normal self-energy $\chi(\mathbf{k}, i\omega_n)$ and the anomalous self-energy $\phi(\mathbf{k}, i\omega_n)$:

$$[1 - Z(\mathbf{k}, i\omega_n)]\omega_n = -\frac{1}{\beta} \sum_{\mathbf{k}', n'} \frac{\omega_{n'} Z(\mathbf{k}', i\omega_{n'}) \lambda(\mathbf{k}, \mathbf{k}', n - n')}{\omega_{n'}^2 Z^2(\mathbf{k}', i\omega_{n'}) + \phi^2(\mathbf{k}', i\omega_{n'}) + [\epsilon_{\mathbf{k}'} + \chi(\mathbf{k}', i\omega_{n'})]^2}, \quad (1)$$

$$\phi(\mathbf{k}, i\omega_n) = \frac{1}{\beta} \sum_{\mathbf{k}', n'} \frac{\phi(\mathbf{k}', i\omega_{n'}) [\lambda(\mathbf{k}, \mathbf{k}', n - n') - V(\mathbf{k} - \mathbf{k}')] }{\omega_{n'}^2 Z^2(\mathbf{k}', i\omega_{n'}) + \phi^2(\mathbf{k}', i\omega_{n'}) + [\epsilon_{\mathbf{k}'} + \chi(\mathbf{k}', i\omega_{n'})]^2}, \quad (2)$$

$$\chi(\mathbf{k}, i\omega_n) = -\frac{1}{\beta} \sum_{\mathbf{k}', n'} \frac{[\epsilon_{\mathbf{k}'} + \chi(\mathbf{k}', i\omega_{n'})] \lambda(\mathbf{k}, \mathbf{k}', n - n')}{\omega_{n'}^2 Z^2(\mathbf{k}', i\omega_{n'}) + \phi^2(\mathbf{k}', i\omega_{n'}) + [\epsilon_{\mathbf{k}'} + \chi(\mathbf{k}', i\omega_{n'})]^2}, \quad (3)$$

where wave vectors \mathbf{k} and \mathbf{k}' lie in the a-b plane(CuO₂ plane) and $\omega_n = (2n + 1)\pi/\beta$ with $\beta = 1/k_B T$. Here the band energy $\epsilon_{\mathbf{k}}$ is measured with respect to the bare band structure chemical potential μ . The Coulomb interaction $V(\mathbf{k} - \mathbf{k}')$ will be neglected, for simplicity, in the following calculation. In the usual phonon-mediated theory, the interaction kernel $\lambda(\mathbf{k}, \mathbf{k}', n - n')$ is written by means of the electron-phonon coupling constant $\bar{g}_{\mathbf{k}, \mathbf{k}', \lambda}$ and the propagator $D_\lambda(\mathbf{k} - \mathbf{k}', i\omega_n - i\omega_{n'})$ of phonons with polarization λ as the following expression,

$$\lambda(\mathbf{k}, \mathbf{k}', n - n') \equiv - \sum_{\lambda} |\bar{g}_{\mathbf{k}, \mathbf{k}', \lambda}|^2 D_\lambda(\mathbf{k} - \mathbf{k}', i\omega_n - i\omega_{n'}), \quad (4)$$

and this is expressed further in terms of the electron-phonon spectral function $\alpha^2 F(\mathbf{k}, \mathbf{k}', \Omega)$ as

$$\lambda(\mathbf{k}, \mathbf{k}', n) = \int_0^\infty d\Omega \frac{2\Omega \alpha^2 F(\mathbf{k}, \mathbf{k}', \Omega)}{(2n\pi k_B T)^2 + \Omega^2}. \quad (5)$$

In this case, $\alpha^2 F(\mathbf{k}, \mathbf{k}', \Omega)$ is positive definite for any value of wave vectors \mathbf{k}, \mathbf{k}' and the phonon energy Ω . But we will consider, from now on, a general electron-boson interaction, therefore, this positive definiteness of $\alpha^2 F(\mathbf{k}, \mathbf{k}', \Omega)$ is not necessarily maintained. Since we assume the square symmetry of the CuO₂ plane by neglecting the anisotropy in the a-b plane, the general interaction spectral function $\alpha^2 F(\mathbf{k}, \mathbf{k}', \Omega)$ is expanded by means of the basis functions $\Psi_i(\mathbf{k})$ of the point group D_{4h} . Here we assume, for simplicity, that the mixing interaction consists of the *s*-wave channel and the *d*-wave channel. Then, in our model, the interaction spectral function is expanded in terms of basis functions $\Psi_s(\mathbf{k}) = 1$ and $\Psi_d(\mathbf{k}) = \cos(\mathbf{k}_x) - \cos(\mathbf{k}_y)$ as follows;

$$\alpha^2 F(\mathbf{k}, \mathbf{k}', \Omega) = \overline{\alpha^2 F_s(\Omega)} \Psi_s(\mathbf{k}) \Psi_s(\mathbf{k}') + \overline{\alpha^2 F_d(\Omega)} \Psi_d(\mathbf{k}) \Psi_d(\mathbf{k}'). \quad (6)$$

Here it should be noted that $\overline{\alpha^2 F_i(\Omega)}$ (*i* = *s*, *d*) are projections of the interaction spectral function into the *s*-channel and the *d*-channel. Therefore, $\lambda(\mathbf{k}, \mathbf{k}', n - n')$ can be also written as

$$\lambda(\mathbf{k}, \mathbf{k}', n) = \lambda_s(n) \Psi_s(\mathbf{k}) \Psi_s(\mathbf{k}') + \lambda_d(n) \Psi_d(\mathbf{k}) \Psi_d(\mathbf{k}'), \quad (7)$$

where

$$\lambda_i(n) = \int_0^\infty d\Omega \frac{2\Omega \overline{\alpha^2 F_i(\Omega)}}{(2n\pi k_B T)^2 + \Omega^2} \quad (i = s, d). \quad (8)$$

Consequently, from the structure of the Eliashberg equations (1)~(3), we can easily see that the functions $Z(\mathbf{k}, i\omega_n)$, $\chi(\mathbf{k}, i\omega_n)$ and $\phi(\mathbf{k}, i\omega_n)$ are also expanded in a following way;

$$Z(\mathbf{k}, i\omega_n) = Z_s(i\omega_n) \Psi_s(\mathbf{k}) + Z_d(i\omega_n) \Psi_d(\mathbf{k}), \quad (9)$$

$$\phi(\mathbf{k}, i\omega_n) = \phi_s(i\omega_n) \Psi_s(\mathbf{k}) + \phi_d(i\omega_n) \Psi_d(\mathbf{k}), \quad (10)$$

$$\chi(\mathbf{k}, i\omega_n) = \chi_s(i\omega_n) \Psi_s(\mathbf{k}) + \chi_d(i\omega_n) \Psi_d(\mathbf{k}). \quad (11)$$

These expansions reduce the Eliashberg equations to the coupled equations for the functions $Z_i(i\omega_n)$, $\chi_i(i\omega_n)$ and $\phi_i(i\omega_n)$ with *i* = *s*, *d*;

$$Z_s(i\omega_n) = 1 + \frac{1}{\omega_n \beta} \sum_{\mathbf{k}', n'} \frac{\omega_{n'} \lambda_s(n - n') [Z_s(i\omega_{n'}) \Psi_s(\mathbf{k}') + Z_d(i\omega_{n'}) \Psi_d(\mathbf{k}')] \Psi_s(\mathbf{k}')}{A(\mathbf{k}', i\omega_{n'})}, \quad (12)$$

$$Z_d(i\omega_n) = \frac{1}{\omega_n \beta} \sum_{\mathbf{k}', n'} \frac{\omega_{n'} \lambda_d(n - n') [Z_s(i\omega_{n'}) \Psi_s(\mathbf{k}') + Z_d(i\omega_{n'}) \Psi_d(\mathbf{k}')] \Psi_d(\mathbf{k}')}{A(\mathbf{k}', i\omega_{n'})}, \quad (13)$$

$$\phi_s(i\omega_n) = \frac{1}{\beta} \sum_{\mathbf{k}', n'} \frac{\lambda_s(n - n') [\phi_s(i\omega_{n'}) \Psi_s(\mathbf{k}') + \phi_d(i\omega_{n'}) \Psi_d(\mathbf{k}')] \Psi_s(\mathbf{k}')}{A(\mathbf{k}', i\omega_{n'})}, \quad (14)$$

$$\phi_d(i\omega_n) = \frac{1}{\beta} \sum_{\mathbf{k}', n'} \frac{\lambda_d(n - n') [\phi_s(i\omega_{n'}) \Psi_s(\mathbf{k}') + \phi_d(i\omega_{n'}) \Psi_d(\mathbf{k}')] \Psi_d(\mathbf{k}')}{A(\mathbf{k}', i\omega_{n'})}, \quad (15)$$

$$\chi_s(i\omega_n) = -\frac{1}{\beta} \sum_{\mathbf{k}', n'} \frac{\lambda_s(n - n') [\chi_s(i\omega_{n'}) \Psi_s(\mathbf{k}') + \chi_d(i\omega_{n'}) \Psi_d(\mathbf{k}') + \epsilon_{\mathbf{k}'}] \Psi_s(\mathbf{k}')}{A(\mathbf{k}', i\omega_{n'})}, \quad (16)$$

$$\chi_d(i\omega_n) = -\frac{1}{\beta} \sum_{\mathbf{k}', n'} \frac{\lambda_d(n - n') [\chi_s(i\omega_{n'}) \Psi_s(\mathbf{k}') + \chi_d(i\omega_{n'}) \Psi_d(\mathbf{k}') + \epsilon_{\mathbf{k}'}] \Psi_d(\mathbf{k}')}{A(\mathbf{k}', i\omega_{n'})}, \quad (17)$$

where

$$A(\mathbf{k}', i\omega_{n'}) \equiv \omega_{n'}^2 [Z_s(i\omega_{n'}) \Psi_s(\mathbf{k}') + Z_d(i\omega_{n'}) \Psi_d(\mathbf{k}')]^2 + [\phi_s(i\omega_{n'}) \Psi_s(\mathbf{k}') + \phi_d(i\omega_{n'}) \Psi_d(\mathbf{k}')]^2 + [\epsilon_{\mathbf{k}'} + \chi_s(i\omega_{n'}) \Psi_s(\mathbf{k}') + \chi_d(i\omega_{n'}) \Psi_d(\mathbf{k}')]^2. \quad (18)$$

In our model, the itinerant electrons are assumed to be described by the nearest-neighbor tight-binding band

$$\epsilon_{\mathbf{k}} = 2\bar{t} [2 - \cos(k_x) - \cos(k_y) - \bar{\mu}], \quad (19)$$

where \bar{t} is the nearest-neighbor hopping energy (taken as 170[meV] in later calculations) and $\bar{\mu}$ is the chemical potential normalized with $2\bar{t}$. These self-consistent coupled equations (12)~(17) are solved numerically. There, the wave number summations over the square Brillouin zone ($-\pi \leq k_i < \pi; i = x, y$) are carried out numerically. The expansion of $\alpha^2 F(\mathbf{k}, \mathbf{k}', \Omega)$ in such a way as given in eq.(6) has already been used in several works [7, 8]. In some work, however, the wave number summation was inverted into an angular integration in k-space assuming the isotropic Fermi surface, and therefore a full summation over the first Brillouin zone was avoided [8].

A final important ingredient to be provided in the calculation is the form of $\overline{\alpha^2 F_i(\Omega)}$ appearing in eq.(6). Even after $\alpha^2 F(\mathbf{k}, \mathbf{k}', \Omega)$ is expanded as in eq.(6), a selective opportunity is still left us in the choice of the functional form for $\overline{\alpha^2 F_s(\Omega)}$ and $\overline{\alpha^2 F_d(\Omega)}$. We may, of course, employ properly two model functions for them different from each other. The numerical results will suffer from a slight modification dependently on the choice of the functional form for them. To investigate closely in what manner they are affected by the detailed form of $\overline{\alpha^2 F_i(\Omega)}$ is a problem in itself worth being studied, however, we postpone it on the next occasion. For our own purpose of this paper, we here make a simple choice for them such that both of $\overline{\alpha^2 F_s(\Omega)}$ and $\overline{\alpha^2 F_d(\Omega)}$ have the same functional form except for multiplicative constants; that is, $\overline{\alpha^2 F_s(\Omega)} = s \overline{\alpha^2 F(\Omega)}$ and $\overline{\alpha^2 F_d(\Omega)} = d \overline{\alpha^2 F(\Omega)}$ under the condition $s + d = 1$. Then we can concentrate on a problem what kind of the symmetry of Cooper-pair can appear when the ratio of the intensity between the s -wave and the d -wave component in the electron-boson spectral function is varied continuously and the band-filling (equivalently, the chemical potential) is done samely. The function $\overline{\alpha^2 F(\Omega)}$, in this case, is quite different in its meaning from the electron-phonon spectral function $\alpha^2 F(\Omega)$ in the conventional Eliashberg equations. As for the functional form of $\overline{\alpha^2 F(\Omega)}$, however, we employ a function which was used in the previous work being expected to model the spectral function in Bismuth-based cuprates [9]. The functional form of this $\overline{\alpha^2 F(\Omega)}$ is shown in Fig.1. It should be mentioned, however, that we tentatively employ this model function merely from a reason that it has comparatively wide spectral distribution as the electronic band has. One may suspect that such a choice of $\overline{\alpha^2 F_i(\Omega)}$ makes a considerable influence on the numerical results and the complex form of $\overline{\alpha^2 F(\Omega)}$ as well as the minute phonon structures of it will affect the conclusions. We will make comments briefly on this point in Section 3.

3 Phase diagrams for the pairing symmetry

For many sets of parameters $(\bar{\mu}, s)$, the reduced chemical potential $\bar{\mu}$ and the mixing parameter s of the s -channel interaction, the Eliashberg equations (12)~(17) are solved at various temperatures. In the calculations, a model function $\alpha^2 F(\Omega)$ is used which is decided to give $T_c = 80[\text{K}]$ for values of parameters $(\bar{\mu}, s) = (1.5, 1.0)$.

At each fixed temperature T , for a given set of values $(\bar{\mu}, s)$ we examine whether the coupled equations (12)~(17) have non-vanishing solutions of the anomalous self-energy $\phi_j(i\omega_n)$ or not. If we find out the solutions $\phi_s(i\omega_n)$ to be non-vanishing for all n , then we recognize that for a given set of $(\bar{\mu}, s)$ the state is superconducting with the s -wave pairing symmetry at this temperature T . If we find out, on the other hand, the solutions $\phi_d(i\omega_n)$ to be non-vanishing, we recognize that the state is superconducting with the d -wave pairing symmetry at the same temperature T . From the symmetry consideration of the form of $\alpha^2 F(\mathbf{k}, \mathbf{k}', \Omega)$ in eq.(6), it is understood that the occurrence of the pairing state with a mixed symmetry is not possible in this case. If we find that both of the solutions $\phi_s(i\omega_n)$ and $\phi_d(i\omega_n)$ vanish for all n , we understand the state to be normal for such values of $(\bar{\mu}, s)$ at the temperature T . At various temperatures, in this way, by changing the values of the parameters $(\bar{\mu}, s)$ we specify the pairing symmetry of the solutions one after the other. We exhibit the resulting diagram in the $\bar{\mu} - s$ plane for the specification of states realized at $T=20[\text{K}]$ and $40[\text{K}]$ in Fig.2(a) and (b), respectively. There, circles and triangles indicate points $(\bar{\mu}, s)$ for which the states are superconducting with the s -wave and the d -wave pairing symmetry, respectively, and squares denote points for which the states are normal at the temperatures. For clarity, the points only in the close proximity of the boundary lines between regions specified by different symmetries are plotted by picking up from a larger number of points examined.

We can easily understand that the boundary line between the normal region and any of the superconducting regions (of s -wave or d -wave) represents just a “equi- T_c ” line, for the points $(\bar{\mu}, s)$ on which the superconductivity (of the s -wave or the d -wave pairing) can be produced with the critical temperature $T_c=20[\text{K}]$ in Fig.2(a) and $T_c=40[\text{K}]$ in Fig.2(b), respectively. In these diagrams, actual calculations are made for $\bar{\mu}$ in a region $0.25 \leq \bar{\mu}$, because the reliability of the calculations becomes worse in the close proximity of $\bar{\mu}=0$: When $\bar{\mu}$ is close to zero, the Fermi energy becomes much less than the spectral width of $\alpha^2 F(\Omega)$, so that the Eliashberg calculation loses its meaning.

Combining the results at $T=20[\text{K}]$ and $40[\text{K}]$ with additional ones at $T=5[\text{K}]$ and $60[\text{K}]$, we make up a phase diagram of the pairing symmetry in the $\bar{\mu} - s$ plane, which is depicted in Fig.3. The region hatched by vertical dotted lines is here after called that of the “ S -phase” where sets of the coordinates $(\bar{\mu}, s)$ can produce s -wave superconductivities. The region hatched by horizontal dotted lines is of the “ D -phase”, which has a corresponding meaning same as in the case of the term “ S -phase”. A thick solid line represents the boundary between regions of different phases. Several thin solid lines drawn together are “equi- T_c ” lines; the value of T_c is indicated on each side of them. The boundary line has not been determined eventually in a region of the phase diagram where $\bar{\mu} < 0.5$ and $s \simeq 0$, because we could not solve the Eliashberg equations at very low temperatures below $5[\text{K}]$ due to the limitations of the computational time. The boundary drawn in a thick dotted line in this region is, therefore, only a guess from the behaviors of the solutions of the Eliashberg equations at $5[\text{K}]$ for some points of $(\bar{\mu}, s)$ in this region.

Looking in this phase diagram in Fig.3, we first notice three points: (i) The d -wave super-

conductivity can hardly occur for small $\bar{\mu}$, even if the d -channel interaction which causes the d -wave pairing is mixed strongly. It seems that there is a lower bound for $\bar{\mu}$ in order for the d -wave superconductivity to be able to appear. (ii) To the contrary, the s -wave superconductivity can occur regardless of the value of $\bar{\mu}$. Even for large $\bar{\mu}$ near $\bar{\mu}=2$ (half-filling case), it can occur if the s -channel interaction is sufficiently strong compared with the d -channel one, i.e. $s/d > 4$. (iii) The boundary curve between the S -phase and the D -phase region has a positive gradient and goes into the upper-half of the phase diagram ($s > 0.5$) when $\bar{\mu} \geq 1.25$. This means that even if the s -channel interaction is mixed largely, the d -wave superconductivity can preferably occur when $\bar{\mu}$ is sufficiently close to the half-filling value 2. And this preference progresses increasingly as $\bar{\mu}$ increases. Reversely, the s -wave superconductivity occurs preferably as $\bar{\mu}$ becomes smaller than 1.25. This preference progresses much more rapidly as $\bar{\mu}$ decreases than the preference of the d -wave superconductivity does as $\bar{\mu}$ increases in the case of the d -wave, since the gradient of the boundary curve is steeper for smaller $\bar{\mu}$.

These characteristic features of the phase diagram are well expected and agree with common feelings. From the calculational point of view, this can be understood as follows. By looking in Eq.(15), for example, of coupled equations (12)~(17), we can see that, in order for $\phi_d(i\omega_n)$ to survive rather than $\phi_s(i\omega_n)$ to do, it is necessary for the value of $\Psi_d(\mathbf{k}')^2$ in the numerators of the summand on the right hand side of the equation to be large compared with the value of $\Psi_s(\mathbf{k}')^2$ around the Fermi surface(line). There, for small $\bar{\mu}$, $\Psi_d(\mathbf{k}')^2$ is much smaller than $\Psi_s(\mathbf{k}')^2 (=1)$, and the effective coupling strength of the d -channel interaction proportional to $\Psi_d(\mathbf{k}')^2$ is entirely overwhelmed by that of the s -channel one. Therefore, the d -wave superconductivity hardly appears in this case. As the value of $\bar{\mu}$ increases and approaches to the half-filling value 2, the maximum value of $\Psi_d(\mathbf{k}')^2$ around the Fermi surface approaches to 4, therefore the average value of $\Psi_d(\mathbf{k}')^2$ around the Fermi surface becomes much higher than the value of $\Psi_s(\mathbf{k}')^2 (=1)$. This produces effectively large coupling strength of the d -channel interaction, which causes the preference of the d -wave superconductivity in the proximity of $\bar{\mu}=2$.

From Fig.3, we can deduce also the behavior of T_c as a function of $\bar{\mu}$ and s . We first notice that “equi- T_c ” curves are distributed densely in the D -phase region while those in the S -phase region are sparse. This means that if the point $(\bar{\mu}, s)$, starting from a some point on the boundary line between the different phase regions, moves away from it along the normal directions of respective “equi- T_c ” lines on both sides of the boundary, the value of $T_c(\bar{\mu}, s)$ increases much more rapidly on the D -phase side than on the S -phase side. In the D -phase region $T_c(\bar{\mu}, s)$ would have the highest value at $(\bar{\mu}, s)=(2,0)$, which would be much larger than the highest value at $(\bar{\mu}, s)=(2,1)$ in the S -phase region. Secondly, from an inspection of the phase diagram, we can find the dependence of T_c on the mixing parameter s for the fixed value of $\bar{\mu}$. In Fig.4, the curves of $T_c(\bar{\mu}, s)$ versus s are shown for several fixed values of $\bar{\mu}$. There, each $T_c - s$ curve has a kink-like minimum point at $s = s_0$ which is the ordinate of the crossing point $(\bar{\mu}, s_0)$ of the line of constant $\bar{\mu}$ and the boundary line between the S -phase and the D -phase region in the phase diagram shown in Fig.3. In the curves for $\bar{\mu}=1.5$ and 0.8, we notice that T_c increases much more rapidly as the d -channel mixing parameter $d = 1 - s$ increases from $d_0 = 1 - s_0$ than it does as the s -channel mixing parameter s increases from s_0 . The magnitude of the gradient of the $T_c - s$ curve in the D -phase region ($s < s_0$) depends on the value of fixed $\bar{\mu}$; the gradient becomes steeper for

larger values of $\bar{\mu}$. On the other hand, in the S -phase region ($s > s_0$) the gradient is almost independent of the value of $\bar{\mu}$. This larger gain of T_c in the D -phase than in the S -phase for a same amount of increase of respective parameters d and s can be understood as follows. In the D -phase region, in addition to the predominance of the d -channel effective coupling compared to the s -channel one, the reduction of T_c by the damping effect owing to the renormalization function $Z(\mathbf{k}, i\omega_n)$ is expected to be small. Because $Z_d(i\omega_n)$ vanishes due to its d -wave symmetry even for large value of d and $Z_s(i\omega_n)$ is small due to smallness of s in the D -phase region, the renormalization function $Z(\mathbf{k}, i\omega_n)$ is small as a whole in the D -phase region. Therefore, T_c is much more sensitive to the change of electron-boson coupling in D -phase than in S -phase. This characteristic behavior of $T_c(\bar{\mu}, s)$ for fixed $\bar{\mu}$ values is also guessed from the estimation by eye the directional derivative of $T_c(\bar{\mu}, s)$ along the line of constant $\bar{\mu}$ in the phase diagram given in Fig.3: The directional derivatives along each line of constant $\bar{\mu}$ changes its sign and is discontinuous at the point $(\bar{\mu}, s_0)$ on the boundary line. The density of the “equi- T_c ” lines along the line of constant $\bar{\mu}$ is much higher in the D -phase region ($s < s_0$) than in the S -phase region ($s > s_0$). And this high density of the “equi- T_c ” line along the line of constant $\bar{\mu}$ in the D -phase region progresses increasingly as the constant $\bar{\mu}$ value becomes larger while it scarcely changes in the S -phase region. Thirdly, we can see that, for a fixed mixing parameter s , $T_c(\bar{\mu}, s)$ increases monotonously as $\bar{\mu}$ does, but the gradient of the $T_c - \bar{\mu}$ curve in the D -phase region is much larger than that in the S -phase region. This larger gradient in the D -phase is also explained from the strong enhancement of the effective electron-boson coupling strength by the increase of $\bar{\mu}$ in the D -phase.

Finally we should mention that the absolute value of T_c itself does not have important meaning in this explanation of the phase diagram. Since in our calculation we employ the model function $\bar{\alpha}^2 F(\Omega)$ having a certain multiplicative constant tentatively chosen, the T_c value can be modified in any way by this multiplicative constant. However, the characteristic feature of the distribution of the “equi- T_c ” lines is expected to remain almost unaltered unless we use a meaninglessly large multiplicative constant. Although the value of T_c beside each “equi- T_c ” line would be scaled by this multiplicative constant, the boundary line which divides the different phase regions would remain almost the same.

Here we would like to make a comment on the effect of the minute structures in the model functions $\bar{\alpha}^2 F_i(\Omega)$ on the numerical results: To put it concisely, the minute structures in $\bar{\alpha}^2 F_i(\Omega)$ do not give any significant and qualitative change to the feature of the results stated above. Since, in the Eliashberg equations (12)~(17), $\bar{\alpha}^2 F_i(\Omega)$ plays its role through the n -dependence of $\lambda_i(n)$ given in eq.(8), the minute structures of $\bar{\alpha}^2 F_i(\Omega)$ are smeared out because of the Lorentzian in the integrand of the right-hand side in eq.(8) so that any serious effect is not produced by their presence. If we employ another forms for $\bar{\alpha}^2 F_i(\Omega)$, of course, results stated above may be modified more or less quantitatively: At finite temperatures, the boundary curve between the normal region and the superconducting region will be modified slightly. The boundary curve between the S -phase and the D -phase region will be also modified in the same way; we have checked this by making additional calculations with a cutoff Lorentzian-type of the model function for $\bar{\alpha}^2 F(\Omega)$. But, in this paper, we do not discuss details of the effect due to the change of the functional form for $\bar{\alpha}^2 F(\Omega)$, because it gives only minor effects to the results in the context of our primary concern as well as it is beyond the purpose of this paper.

4 Conclusions

We have solved the two-dimensional Eliashberg equations employing an electron-boson interaction which has the s -wave channel and the d -wave channel. The electronic state is assumed to be described by a nearest neighbor tight-binding band. We examined what kind of symmetry of the superconducting state could appear when the chemical potential $\bar{\mu}$ and the mixing parameter s of the s -channel interaction (equivalently, the mixing parameter $d = 1 - s$ of the d -channel interaction) are varied. The result was summarized into a phase diagram for the pairing symmetry in the $\bar{\mu} - s$ parameter plane. The boundary line between the s -wave and the d -wave pairing region in the $\bar{\mu} - s$ parameter plane has been determined except for the region where $\bar{\mu}$ is less than 0.5 and $s(d)$ is very small(large). It seems that there is a lower bound in $\bar{\mu}$ for the occurrence of the d -wave superconductivity. Because of the limitation of the computational time, we could not give a decisive answer to the question whether there is the possibility for the occurrence of the d -wave superconductivity when $\bar{\mu}$ is small and $d(= 1 - s)$ is very large. But we are convinced, from several trial calculations at very low temperature, that T_c would be extremely low if the d -wave superconductivity could ever occur for $(\bar{\mu}, s)$ in such a region. Therefore, we can almost say that there is a threshold value of $\bar{\mu}$ below which the d -wave superconductivity hardly occurs even when the d -channel interaction is mixed strongly. On the other hand, the s -wave superconductivity can occur for any value of $\bar{\mu}$: Even for large $\bar{\mu}$ near $\bar{\mu} = 2$ (half-filling case), it can occur if the s -channel interaction is sufficiently strong compared with the d -channel one.

Although the area of the d -wave pairing in the $\bar{\mu} - s$ plane is relatively small and spread only around the corner $(\bar{\mu}, s) = (2, 0)$, once the d -wave superconductivity occurs, it holds the advantage of getting a high T_c over the s -wave superconductivity. In the close vicinity of half-filling ($\bar{\mu} = 2$), the effective coupling strength of the d -channel interaction is strongly enhanced and T_c goes up rapidly as d increases with a fixed $\bar{\mu}$ and it does the same as $\bar{\mu}$ increases with a fixed $d(= 1 - s)$. Within our model calculation, therefore, the case of a pure d -channel interaction and half-band filling is the most advantageous one so as to achieve a high T_c superconductivity.

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Figure captions

Fig.1. Model function of $\overline{\alpha^2 F(\Omega)}$.

Fig.2. Diagram for the specification of states (a) at 20[K] and (b) at 40[K]. Circles and triangles represent points $(s, \bar{\mu})$ for which the states are superconducting with the s -wave pairing symmetry and with the d -wave one, respectively, and squares indicate points for which the states are normal at 20[K] in (a) and at 40[K] in (b).

Fig.3. Phase diagram of the pairing symmetry in the $\bar{\mu} - s$ plane. The region hatched by vertical dotted lines is that of the s -wave symmetry where sets of the coordinates $(\bar{\mu}, s)$ can produce s -wave superconductivities. The region hatched by horizontal dotted lines is that of the d -wave symmetry. A thick solid line represents the boundary between regions of the s -wave symmetry and of the d -wave one. Several thin solid lines drawn together are equi- T_c lines, on each side of which the value of T_c is indicated.

Fig.4. Curves of T_c versus s for fixed $\bar{\mu}$ values, 1.5, 0.8 and 0.5. In each curve, the part drawn in solid line exhibits T_c values of the s -wave superconductivity and the part in dashed line does T_c values of the d -wave superconductivity.